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STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART I DISCRETE MODELS

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STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART I DISCRETE MODELS

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IS 9300 (Part 1) : 1979 STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART 1 DISCRETE MODELS

(*Page* 5, *clause* 3.1.1, *last line*) — Substitute 'or their collection is known as event' *for* 'are known as events'

(*Page* 5, *clause* 3.1.2) — Substitute the following for the existing clause:

'3.1.2 *Exhaustive Event* — The collection of all possible outcomes in any trial is known as exhaustive event '

(*Page* 5, *clause* 3.1.4) — Substitute the following for the existing clause.

'3.1.4 *Favourable Event* — The set of outcomes favourable to an event in a trial is the set of all outcomes which entail the happening of the event.'

(*Page* 5, *clause* 3.1.6) — Substitute the following for the existing clause:

'3.1.6 *Independent Events* — Several events are said to be independent if the chance of occurrence (or non-occurrence) of an event is not affected by the supplementary knowledge concerning the occurrence (or non-occurrence) of any of the remaining events '

(*Page* 6, *clause* 3.3.1) — Shift the matter of 3.4 at the end of this clause.

(*Page* 7, *clause* 3.3.2) — Shift the matter of 3.5 at the end of this clause.

(*Page* 7, *clause* 3.4) — Delete '3.4'

(*Page* 8, *clause* 3.5) — Delete '3.5'.

(*Page* 11, *clause* 5.1.1, last line) — Substitute '$\left(\frac{n}{2}\right) q^{n-2}p^{2}$' *for* '$\left(\frac{n}{2}\right) q^{n-2} p^{2}$'.

(*Page* 15, *clause* 5.6) — Substitute the following for the existing equation

$$\text{`}\int_{x_i-\frac{1}{2}}^{x_i+\frac{1}{2}} N\left[\,np,\ \sqrt{np(1-p)}\,\right] dx = \int_{\frac{x_i-np-\frac{1}{2}}{\sqrt{np(1-p)}}}^{\frac{x_i-np+\frac{1}{2}}{\sqrt{np(1-p)}}} N(0,\ 1)\, dx\text{'}$$

(*Page* 16, *clause* 6.3) — Insert the following at the end of this clause:

'**Covariance** — The covariance between the i^{th} and the j^{th} possible outcomes is obtained as:

$$Cov\ (x_i,\ x_j) = \sqrt{np_i\, p_j}\text{'}$$

(*Page* 22, *clause* 8.2, *line* 3) — Substitute the following for the existing formula for standard deviation:

$$\text{`}\sqrt{v(r)} = \left[\frac{nD}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)\right]^{\frac{1}{2}}\text{'}$$

(*Page* 22, *clause* 8.2.1, *line* 5) — Substitute the following for the existing equation:

$$\text{`Standard deviation} = \sqrt{25\left(\frac{16}{200}\right)\left(1-\frac{16}{200}\right)\times\frac{200-25}{200-1}}\text{'}$$

(*Page* 29, *row* 21, *col* 10) — Substitute '0 7361' *for* '0 7461'

(MSD 3)

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STATISTICAL MODELS FOR INDUSTRIAL APPLICATIONS

PART I DISCRETE MODELS

0. FOREWORD

0.1 This Indian Standard (Part I) was adopted by the Indian Standards Institution on 12 September 1979, after the draft finalized by the Quality Control and Industrial Statistics Sectional Committee had been approved by the Executive Committee.

0.2 Statistical models have been found to be of immense help in different industries. The behaviour of various characteristics in industrial processes may often be well explained by many statistical distributions to which they can reasonably be approximated. For example, the number of defectives in the case of a manufactured item may follow the binomial distribution, whereas the number of defects per unit in a finished product may follow Poisson distribution. The theoretical distributions like binomial, Poisson and exponential may readily be used in many cases for studying the behaviour of various characteristics of manufacturing processes in the industry.

0.3 To facilitate easy application, this standard on statistical models is being published in two parts. Whereas the discrete models are covered in this part, the continuous models would be considered in Part II.

0.4 This standard is one of a series of Indian Standards pertaining to statistical quality control and acceptance sampling. Other standards published so far in the series are given on fourth cover page.

0.5 In reporting the result of a test or analysis made in accordance with this standard, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960*.

1. SCOPE

1.1 This standard describes certain discrete statistical models, their potentiality and application in industries with suitable illustrations.

---

*Rules for rounding off numerical values (*revised*).

1.1.1 The models covered in this standard are those of binomial, multinomial, Poisson and hypergeometric.

2. TERMINOLOGY

2.0 For the purpose of this standard, the following definitions shall apply.

2.1 Item — Ultimate unit of product or material on which inspection will be performed.

2.2 Population — The totality of items under consideration.

2.3 Characteristic — A property which helps to differentiate between the items of a given population. The differentiation may be either quantitative (by variables) or qualitative (by attributes).

2.4 Relative Frequency — The ratio of the number of times a particular value (or a value falling within a given class interval) is observed to the total number of observations.

2.5 Frequency Distribution — The relationship between the values of a characteristic and their absolute or relative frequencies. The distribution is often presented as a table with special groupings (class intervals) if the values are measured on a continuous scale.

2.6 Arithmetic Mean (Average) — Sum of the values of the observations divided by the number of observations.

2.7 Variance — A measure of dispersion based on the mean square deviation from the arithmetic mean.

2.8 Sample — Group of items drawn from a lot for inspection.

2.9 Sample Size — Number of items in the sample.

2.10 Defective — An 'item' the quality of which does not meet the specified requirements.

2.11 Fraction Defective — The number of defective items divided by the total number of items.

2.12 Percent Defective — The fraction defective multiplied by 100.

2.13 Degrees of Freedom — The number of independent component values which are necessary to determine a statistic.

2.14 Goodness of Fit — A measure of the agreement between an observed distribution and a theoretical distribution, specified a priori or fitted to the observations.

2.15 Observed Value — The value of a characteristic determined as a result of an observation or test.

2.16 Null Hypothesis — The hypothesis (or assumption) of the equivalence (or no difference) between the effects of method(s) so that the sample(s) emanates from the same lot.

2.17 Level of Significance — The probability of rejecting the null hypothesis when it is true. Conventionally it is taken to be 5 percent or 1 percent.

2.18 Statistic — A function of observed values derived from the sample.

3. GENERAL CONCEPTS

3.1 The concept of probability is essential in understanding the statistical distributions or models. Probability is effectively the mathematical statement of chance. It may be either:

a) ' a priori ' consideration, or
b) ' frequency consideration ' or ' empirical definition '.

But before defining probability, it is necessary to become familiar with various terms used in the definition.

3.1.1 *Trial and Event* — If an experiment repeated under identical conditions does not give unique results but different outcomes, then the experiment is known as a trial and the outcomes are known as events.

3.1.2 *Exhaustive Events* — The total number of possible outcomes in any trial is known as exhaustive events.

3.1.3 *Mutually Exclusive Events* — If an occurrence of any one of the events precludes the occurrence of all others, then the events are said to be mutually exclusive.

3.1.4 *Favourable Event* — The number of cases favourable to an event in a trial is the number of outcomes which entail the happening of the event.

3.1.5 *Equally Likely Events* — The outcomes of a trial are said to be equally likely, if taking into consideration all the relevant evidence, there is no reason to expect one in preference to other.

3.1.6 *Independent Events* — Several events are said to be independent if the happening (or non-happening) of an event is not affected by the supplementary knowledge concerning the occurrence of any number of remaining events.

3.2 Definitions of Probability

3.2.1 *'A Priori' Definition* — 'A priori' definition is based on the prior information. For example, without tossing a coin it is known that there are two possible outcomes of head or tail if the coin is tossed. Both these outcomes are mutually exclusive and exhaustive and equally likely. Only one case is favourable to appearance of either head or tail. Thus, if a trial results in n exhaustive, mutually exclusive and equally likely cases and out of which m cases are favourable to an event E, then the probability of happening of E denoted by $P(E)$ or p where,

$$P(E) = p = \frac{m}{n}$$

3.2.2 *Frequency or Empirical Definition* — Suppose a trial is conducted N times, out of which n_1 times an event E has occured. Therefore $\frac{n_1}{N}$ is the proportion of times that the event has occurred. By repeating this experiment with large and larger values of N, this proportion will stabilize to a certain value and this stabilized value is the probability of the event.

If $P(E) = \frac{\text{Total number of occurences of the event}}{\text{Total number of trials}}$

then the limit of $P(E)$ as $N \to \infty$ is the probability 'p' of the event, obviously $P(E)$ lies between 0 and 1.

$P(E) = 1$ means that the event is a certainity and $P(E) = 0$ means the event is impossible.

3.3 Laws of Probability — The two laws of probability are:

a) additive (or total probability) law,
b) multiplicative (or compound probability) law.

3.3.1 *Additive Law* — If an event E occurs when any one of the set of mutually exclusive events E_1, E_2, ... E_k happens then the probability of $P(E)$ is given by:

$$P(E) = P(E_1) + P(E_2) + \dots\dots\dots + P(E_k)$$

This is known as the additive or total probability law.

3.3.2 *Multiplicative Law* — This law governs the joint occurrence of events. Let us take one event E to be the joint occurrence of events E_1, E_2, ... E_K. Now joint occurrence of these events can be either

a) independent, or

b) otherwise; as E_1 occurs, E_2 occurs depending on the occurrence of E_1, E_3 occurs depending on the occurrences of E_1 and E_2 .. ... lastly E_K occurs depending on the occurrences of E_1, E_2...... E_{K-1}.

Thus mathematically, this law states that:

i) $P(E) = P(E_1)\, P(E_2) \;......\; P(E_K)$ for independent events, and

ii) $P(E) = P(E_1)\, P(E_2/E_1)\, P(E_3/E_1E_2) \;......\; P(E_K/E_1E_2 \;..\; ...\; E_{K-1})$
Where $P(E_2/E_1)$ means the conditional probability E_2 after E_1 has occurred and so on.

3.4 Example — Let the supplies of 5 000 and 3 000 components received from two firms X and Y have the following break-up of quality:

	Number of Components		*Total*
	Supplied By X	Supplied By Y	
Having a major defect	140	60	200
Having a minor defect	400	150	550
Having no defects	4 460	2 790	7 250
Total	5 000	3 000	8 000

The probability of getting a component with a major defect from the entire supply $= \frac{200}{8\,000} = 0{\cdot}025$.

Knowing that the component is supplied by X, the conditional probability of getting a component with major defect $= \frac{140}{5\,000} = 0{\cdot}028$.

Knowing that the component is supplied by Y, the conditional probability of getting a component with major defect $= \frac{60}{3\,000} = 0{\cdot}020$.

Hence the probability of getting a component with major defect from the entire supply = (probability of getting a component from X) × (conditional probability of it having a major defect) + (probability of getting a component from Y) × (conditional probability of it having a major defect)

$$= \frac{5\,000}{8\,000} \times 0{\cdot}028 + \frac{3\,000}{8\,000} \times 0{\cdot}020$$

$$= 0{\cdot}625 \times 0{\cdot}028 + 0{\cdot}375 \times 0{\cdot}020$$
$$= 0{\cdot}017\,5 + 0{\cdot}007\,5$$
$$= 0{\cdot}025$$

3.5 Example — Three lots of manufactured items contain 2, 5 and 3 percent of defectives respectively. One item is selected from each of the lot. What is the probability that selected items show 2 defectives and 1 non-defective ?

The above example demonstrates the application of both the laws of probability. Denoting defectives by B (bad item) and non-defectives by G (good item), it can be seen that 2 defectives and one non-defective can come from:

	Case I	Case II	Case III
Lot 1	B	B	G
Lot 2	B	G	B
Lot 3	G	B	B

The probability for case I is the probability for joint occurrence of B, from Lot 1, B from Lot 2 and G from Lot 3 (multiplicative law) and similarly for cases II and III.

Now the probability of obtaining a defective B from Lot 1 is

$$P_1(B) = 0{\cdot}02$$

and probability of non-defective from Lot 1 is

$$P_1(G) = 0{\cdot}98$$

Similarly $P_2(B) = 0{\cdot}05$, $P_2(G) = 0{\cdot}95$ for Lot 2

and $P_3(B) = 0{\cdot}03$, $P_3(G) = 0{\cdot}97$ for Lot 3

Thus probability for cases I, II and III are:

Case I — $P_1(B)\,P_2(B)\,P_3(G) = 0{\cdot}02 \times 0{\cdot}05 \times 0{\cdot}97 = 0{\cdot}000\,97$

Case II — $P_1(B)\,P_2(G)\,P_3(B) = 0{\cdot}02 \times 0{\cdot}95 \times 0{\cdot}03 = 0{\cdot}000\,57$

Case III — $P_1(G)\,P_2(B)\,P_3(B) = 0{\cdot}98 \times 0{\cdot}05 \times 0{\cdot}03 = 0{\cdot}001\,47$

Since the event of appearance of 2 defectives and 1 non-defective is possible when any one of the three cases happens, the probability of the event will be the sum of the probabilities for cases I, II and III.

Thus the probability of obtaining 2 defectives and 1 non-defective is:

$$0{\cdot}000\,97 + 0{\cdot}000\,57 + 0{\cdot}001\,47 = 0{\cdot}003\,01$$

This probability is quite small. Hence, occurrence of such an event is very rare. Even from common sense it is evident that the possibility of getting 2 defectives out of 3 samples drawn from 3 different lots, each having very few defectives is quite remote.

4. PROBABILITY DISTRIBUTIONS

4.1 Suppose a sample of n items is drawn from a lot of manufactured items and the number of defectives in the sample are noted. The defectives in the sample can be from 0 to n. If N sets of such samples are taken then we get frequency of samples having 0, 1, 2.........n defectives. Let i denote the number of defectives in the sample and f_i the frequency of the samples having i defectives. It is then possible to prepare the following table:

TABLE 1 THE FREQUENCY OF DEFECTIVES

i (1)	f_i (2)	Relative Frequency f_i/N (3)	Probability (4)
0	f_0	f_0/N	p_0
1	f_1	f_1/N	p_1
2	f_2	f_2/N	p_2
.			
.			
.			
.			
n	f_n	f_n/N	p_n
Total	$\Sigma f_i = N$	$\frac{\Sigma f_i}{N} = 1$	$\Sigma p_i = 1$

4.2 The third column gives the relative frequency of samples having 0, 1, 2............... n defectives. By the frequency definition of probability, these relative frequencies can be taken as the probabilities $p_0, p_1, ...p_n$ of x_i taking 0, 1, 2 ....n values.

4.2.1 Thus, in general if x takes values x_1, x_2.........x_n with probabilities p_1, p_2.........p_n, the assembly of x_i's with probabilities p_i constitute the probability distribution of the variable x. Since x takes only discrete values, the probability distribution is a discrete one. Any variable which takes different values with different probabilities is called a random or stochastic variable.

4.2.2 Since x takes all possible values with one or other probability p_i obviously $\Sigma p_i = 1$.

4.3 Mean and Variance of Probability Distribution — Just like the frequency distribution we can calculate the mean and variance of a probability distribution. The mean of the probability distribution is called the expected value of the variable x and denoted by $E(x)$. The variance of the probability distribution is denoted by $V(x)$.

4.3.1 If x takes values $x_1, x_2, \ldots\ldots\ldots x_n$ with probabilities $p_1, p_2 \ldots\ldots p_n$ then:

$$E(x) = \sum_{i=1}^{n} p_i x_i$$

$$\text{and } V(x) = \sum_{i=1}^{n} p_i \left[x_i - E(x) \right]^2$$

$$= \sum_{i=1}^{n} p_i x_i^2 - \left[E(x) \right]^2$$

5. BINOMIAL MODEL

5.1 Consider the case of a production line inspection system in which items are inspected. Each inspection can have only two outcomes, whether the item is defective (B) or non-defective (G). The probability of getting a defective item is assumed to be constant, say p. Naturally the probability of a non-defective item is $q = (1 - p)$ by the law of probability. Let us now take a sample of 3 items for inspection. The number of defectives say r in the sample can be 0, 1, 2 or 3. They can occur in several ways in the sample and are shown below along with their probabilities:

No. of Defectives, r	*Sample*			*Probability*
	1st item	2nd item	3rd item	
0	G	G	G	q^3
1	G	G	B	$3q^2p$
	G	B	G	
	B	G	G	
2	G	B	B	$3qp^2$
	B	G	B	
	B	B	G	
3	B	B	B	p^3

5.1.1 Obviously total probability ($q^3 + 3q^2p + 3qp^2 + p^3$) = $(q+p)^3$ = 1 and the probabilities for $r = 0, 1, 2$ and 3 are the terms of the binomial expansion of $(q + p)^3$. Accordingly for sample of size n, the probabilities for $r = 0, 1, 2 \ldots . n$ are the terms of the expansion $(q + p)^n$ that is $q^n, \binom{n}{1} q^{n-1} p, \binom{n}{2} q^{n-2}p^2, \ldots\ldots \binom{n}{n} p^n$ respectively.

5.1.2 In general, the probability of r defectives and $(n - r)$ non-defectives is $\binom{n}{r} p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r}$

$$\text{and} \quad \sum_{r=0}^{n} \binom{n}{r} p^r q^{n-r} = 1$$

Thus when n independent trials are conducted, each trial resulting in only two outcomes and the probability of success in each trial remaining constant, the probability distribution of number of success is said to follow binomial law. In the example considered above the distribution of r in a sample of n pieces which can be taken as equivalent to n independent trials follows binomial law.

5.2 Cumulative Probability for Binomial Model

5.2.1 In binomial model, the probability of r defectives and $(n - r)$ non-defectives is given by:

$$\binom{n}{r} p^r q^{n-r}$$

Hence, the cumulative probability of having 0, 1, 2… . up to x defectives is given by the expression:

$$\sum_{r=0}^{x} \binom{n}{r} p^r q^{n-r}$$

Varying x from 0 to n, one can get different cumulative probabilities.

For obtaining the individual and cumulative probabilities of binomial distribution a reference is invited to Appendix A and B respectively.

5.2.2 *Example* — A welded pipeline has a mean of five welds every 100 metres. The probability of a weld being defective is 0·05. What are the probabilities that:

a) no welds (per 100 m) are defective,

b) less than two (per 100 m) are defective, and

c) at least four (per 100 m) will pass inspection.

Here the probability of a defective weld = 0·05

and probability of a non-defective weld = 0·95

a) Probability that no welds are defective $= (0{\cdot}95)^5$
$= 0{\cdot}77$

b) Probability of less than 2 defectives = probability (no defective) + probability (1 defective)
$= (0{\cdot}95)^5 + 5(0{\cdot}95)^4(0{\cdot}05) = 0{\cdot}98$

c) Probability (at least 4 passes) = probability (no defective) + probability (1 defective)
= same as (b) = 0·98

5.2.3 *Example* — The probability of a single article being defective in a production process is 0·02. A sample of 10 is taken every time for inspection. At most how many defectives should be allowed if it is desired that 98 percent of the samples should be passed by inspection?

Here $p = 0{\cdot}02$ and $n = 10$

Assuming that at most x defectives may be allowed, the probability of x defectives should be equal to 0·98, that is,

$$\sum_{r=0}^{x} \binom{10}{r} (0{\cdot}02)^r (0{\cdot}98)^{10-r} = 0{\cdot}98$$

where r is the number of defectives.

From the Appendix B we get the value of $x = 1$ corresponding to $p = 0{\cdot}02$ and $n = 10$. Thus at most 1 defective can be allowed.

5.3 Mean and Variance

$$\text{Mean} = E(r) = \sum_{r=0}^{n} r \binom{n}{r} p^r q^{n-r} = np$$

$$\text{Variance} = E[r - E(r)]^2 = \sum_{r=0}^{n} (r-np)^2 \binom{n}{r} p^r q^{n-r}$$
$$= npq$$

$$\text{Standard deviation} = \sqrt{npq}$$

5.3.1 *Example* — From a process producing 10 percent defectives, 30 components are selected at random. What is the expected number of defectives and the standard deviation?

$$\text{Expected number of defectives} = 30 \times \frac{10}{100} = 3$$

$$\text{Standard deviation} = \sqrt{30 \times \frac{10}{100}\left(1 - \frac{10}{100}\right)}$$

$$= \sqrt{30 \times \frac{10 \times 90}{100 \times 100}}$$

$$= \sqrt{2 \cdot 7}$$

$$= 1 \cdot 64$$

5.4 Fitting of Binomial Model — It has been mentioned in **4.1** that relative frequencies for various values of the variate from the observed data can be taken as the probabilities of the variate taking these values. Thus for fitting a binomial model to a set of data, it is necessary to calculate the theoretical frequencies corresponding to the observed frequencies. The theoretical frequencies are calculated on the basis of the binomial distribution fitted to the data. The following example will clearly show the different steps in fitting the binomial model.

5.4.1 *Example* — The distribution of 50 samples of 100 items each according to number of defective items observed is given below:

No. of Defective Items (*r*)	*No. of Samples* (*frequency f*)
0	8
1	12
2	13
3	9
4	7
5	0
6	1
7	0
	50

Fit a binomial model to the above data. From the data, mean number of defectives per sample

$$= \frac{(0 \times 8 + 1 \times 12 + 2 \times 13 + \ldots\ldots\ldots\ldots\ldots 7 \times 0)}{50}$$

$$= 1 \cdot 98$$

and therefore proportion of defectives in sample = 1·98/100 = 0·019 8 ≈ 0·02 as the sample size is 100.

Taking this proportion of ·02 as the probability of a defective the binomial model can be set up. Thus the probability of r defectives and (n — r) nondefectives in a sample of 100 is given by:

$$\binom{100}{r} (0\cdot02)^r (0\cdot98)^{100-r} = P(r)$$

Putting $r = 0, 1, 2$...... the probabilites $P(0), P(1), P(2)$........ , that is, the probabilities of 0, 1, 2......... defectives in the sample are obtained. Multiplying these probabilities by 50 we obtain the expected number of samples having 0, 1, 2 .... defectives as follows:

TABLE 2 OBSERVED AND EXPECTED FREQUENCIES OF SAMPLES

NUMBER OF DEFECTIVE ITEMS (r) (1)	NUMBER OF SAMPLES Observed (2)		NUMBER OF SAMPLES Expected (3)	
0	8		6·6	
1	12		13·5	
2	13		13·7	
3	9		9·1	
4	7	8	4·5	7·1
5	0		1·8	
6	1		0·6	
7	0		0·2	
TOTAL	50		50	

5.5 Goodness of Fit From χ^2 - Test — After calculating the expected frequencies for different number of defectives, their closeness to the observed frequencies are tested with the help of

$$\chi^2 = \Sigma \frac{(\text{ observed } - \text{ expected })^2}{\text{expected}} \text{ after pooling the adjacent classes,}$$

so as to make the expected frequency of each class at least 5. For the example given in **5.4.1** the calculated value of χ^2 which is 0·61 is much less than 7·82, the tabulated value of χ^{2*} for 3 = (5 − 1 − 1) degrees of freedom, at the 5% level of significance. Hence it is concluded that binomial is a good fit to the data.

5.6 The binomial model discussed earlier may be approximated by a suitable normal model†. If n and p are parameters of the binomial model, this can be approximated by a normal model with mean $= np$ and standard deviation $= \sqrt{np\,(1-p)}$ and the probability of getting x_1 successes in n trails is approximated by

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**See* Appendix A of IS : 6200 (Part II)-1977 'Statistical test of significance, Part II χ^2 - test (*first revision*) '.

†*See* Clause 9 of IS : 7200 (Part I)-1974 'Presentation of statistical data: Part I Tabulation and summarization '.

$$\int_{x_1-\frac{1}{2}}^{x_1+\frac{1}{2}} \mathcal{N}\left[np, \sqrt{np\,(1-p)}\,\right] dx = \int_{\frac{x_1-np-\frac{1}{2}}{\sqrt{np\,(1-p)}}}^{\frac{x_1-np+\frac{1}{2}}{\sqrt{np\,(1-p)}}} \mathcal{N}\,(0, 1)\,dx$$

NOTE — For $x_1 = 0$, the integration is from $-\infty$ to $\frac{1}{2}$ whereas for $x_1 = n$ the integration is from $n-\frac{1}{2}$ to ∞.

6. MULTINOMIAL MODEL

6.1 The multinomial model is a generalisation of binomial model. This model refers to n independent trials of an identical random process in which there are k possible outcomes, unlike binomial model where only two outcomes are possible.

6.2 Let p_i be the probability of ith outcome. Thus $\sum\limits_{i=1}^{k} p_i = 1$. Also let x_i be the numbers of trials in which ith outcome has occurred. Obviously $\Sigma x_i = n$. The multinomial law gives the probability of a particular set of values of the x_i regardless of the order in which the various outcomes occur. Thus the model can be represented in the following tabular form:

Possible Outcomes	*Frequency of Occurrences*	*Probability*
1	x_1	p_1
2	x_2	p_2
.	.	.
.	.	.
.	.	.
.	.	.
k	x_k	p_k
Total	n	1

The probability function for this law is thus defined as:

$$= \frac{n\,!}{x_1\,!\; x_2\,! \;\ldots\ldots\ldots\; x_k\,!}\; p_1^{\,x_1} \ldots\ldots\ldots p_k^{\,x_k}$$

The probabilities of different combinations of outcomes from n independent trials are given by the various terms in the expansion of the multinomial $(p_1 + p_2 \ldots\ldots\ldots + p_k)^n$

6.2.1 This distribution is useful in industry when different grades of a product have to be judged.

6.3 Mean and Variance — The mean and variance of the ith possible outcome are obtained as:

$$E (x_i) = np_i$$

$$V (x_i) = np_i q_i$$

Standard deviation $= \sqrt{np_i \, q_i}$

6.4 Example — The item produced by a production process is classified as overweight, acceptable and underweight, the probabilities for which are 0·2, 0·7, and 0·1 respectively. In a sequence of 10 independent trials calculate the probability of 1 overweight item, 6 acceptable items and 3 under weight items.

By applying the multinomial law we get the required probability as

$$\frac{10\,!}{1\,!\;6\,!\;3\,!} (0{\cdot}2)^1 (0{\cdot}7)^6 (0{\cdot}1)^3$$

$$= 0{\cdot}02$$

7. POISSON MODEL

7.1 Calculations based on binomial distribution are sometimes very laborious specially when the number of terms involved are many and number of items, n is very large. A simple approximation may be obtained to any term of binomial and this approximation is called Poisson exponential binomial limit or briefly Poisson law or Poisson model. The larger the value of n and smaller the value of p, the more close is the Poisson approximation. Thus when n tends to infinity (very large) and p is very small but np is finite and constant, one can use the Poisson model.

The probability function of Poisson model can be written as:

$$\frac{e^{-\mathrm{m}} m^r}{r\,!} , 0 \leq r \leq \infty$$

where m is the mean or expected number of successes (defectives) and r is the number of successes. The probabilities of 0 success, 1 success, 2 successes ……… etc are given by:

$$e^{-m},\ m_{e}^{-m},\ \frac{m^2 e^{-m}}{2\,!} \ldots\ldots$$

Again

$$\sum_{r=0}^{\infty} e^{-m} \frac{m^r}{r\,!} = 1$$

7.1.1 *Cumulative Probability for Poisson Model* — In **7.1** it has been shown that the probability of having r successes for Poisson model is $e^{-m} \frac{m^r}{r\,!}$. The cumulative probability of having 0, 1, 2 .. .. up to x successes (defectives) is therefore:

$$\sum_{r=0}^{x} e^{-m} \frac{m^r}{r\,!}$$

x varying from 0 to ∞, one can get different cumulative probabilities. For individual terms of Poisson and cumulative terms a reference is invited to Appendices C and D respectively.

7.1.2 *Example* — In a certain factory turning out razor blades, there is a small chance of 0·002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson model to calculate the approximate number of packets containing no defective, one defective, two defectives and at least two defectives respectively in a consignment of 10 000 packets.

The mean number of defective blades per packet:

$$= 10 \times 0{\cdot}002 = 0{\cdot}02$$

Poisson model for r defectives in this case will be:

$$\frac{e^{-{\cdot}02} ({\cdot}02)^r}{r\,!}$$

With the help of Appendix C, it is obtained that

$$\text{probability of no defective blade} = \frac{e^{-0{\cdot}02} (0{\cdot}02)^0}{0\,!} = 0{\cdot}980\,2$$

$$\text{probability of one defective blade} = \frac{e^{-0{\cdot}02} (0{\cdot}02)^1}{1\,!} = 0{\cdot}019\,6$$

$$\text{probability of 2 defective blades} = \frac{e^{-0{\cdot}02} (0{\cdot}02)^2}{2\,!} = 0{\cdot}000\,2$$

probability of at least 2 defective blades = Probability of 2 defective + Probability of 3 defectives + ...

= 1 − Probability of no defective − Probability of 1 defective

= 1 − 0·980 2 − 0·019 6

= 1 − 0·999 8 = 0·000 2

Thus number of packets having:

i) no defective	= 0·980 2 × 10 000 =	9 802
ii) one defective	= 0·019 6 × 10 000 =	196
iii) two defectives	= 0·000 2 × 10 000 =	2
iv) at least two defectives	= 0·000 2 × 10 000 =	2

7.1.3 *Example* — Find the probabilty that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.

Here mean $m = 200 \times 0{\cdot}02 = 4$

The required probabilty $= \sum_{r=0}^{5} e^{-4} \frac{4^r}{r\,!}$

$= 0{\cdot}785$

7.2 Mean and Variance — It has been shown (*see* **7.1**) that m in the Poisson model is the mean of the distribution.

$$\text{Thus mean} = \sum_{r=0}^{\infty} r\, e^{-m} \frac{m^r}{r\,!} = m$$

$$\text{and variance} = \sum_{r=0}^{\infty} (\, r - m\,)^2 e^{-m} \frac{m^r}{r\,!} \doteq m.$$

Standard deviation $= \sqrt{m}$

for Poisson model, the mean and variance are the same.

7.3 Fitting a Poisson Model — It has already been shown (*see* **7.2**) that the property of Poisson distribution is that its mean and variance are same. Thus it is necessary to examine the mean and variance of the experimental data before fitting a Poisson model to it. If they vary widely Poisson model will not be a proper fit. In case the difference between mean and variance is of small order, this model should be tried and a proper goodness of fit test be carried out.

7.3.1 *Example* — The following data gives the number of defective screws in a box of 100 screws for 400 boxes:

No. of defective screws per box of 100 screws(r)	0	1	2	3	4	5	6	7	8	9	10	Total
No. of boxes	103	143	98	42	8	4	2	0	0	0	0	400

Fit a Poisson model and carry out the goodness of fit.

Firstly calculate:

$$\text{Mean} = 1{\cdot}32$$

$$\text{and Variance} = 1{\cdot}30$$

Since mean and variance are approximately same, a Poisson model can be fitted. The expected frequencies are calculated from the expression:

$$400\,\frac{e^{-1{\cdot}32}\,(\,1{\cdot}32\,)^{r}}{r\,!}$$

by putting $r = 0, 1, 2, \ldots\ldots\ldots\ldots\ldots 10$. These are

107, 141, 93, 41, 13, 4, 1, 0, 0, 0, 0

The various steps in the goodness of fit test are shown in Table 3.

TABLE 3 COMPUTATION OF GOODNESS OF FIT — χ^2 TEST

No. of Defective Screws per Box (r)	Observed Frequency (O)		Expected Frequency (E)		$O-E$	$(O-E)^2$	$(O-E)^2/E$
(1)	(2)		(3)		(4)	(5)	(6)
0	103		107		−4	16	0·15
1	143		141		2	4	0·03
2	98		93		5	25	0·27
3	42		41		1	1	0·02
4	8		13		5	25	1·92
5	4	6	4	5	1	1	0·20
6	2		1				
7	0		0				
8	0		0				
9	0		0				
10	0		0				
Total	400		400				2·59

Since there are 6 classes after pooling and two constraints, one for the total frequency and the other for the mean, the available degrees of freedom (df) = 4. The tabular value* of χ^2 for 4 df at 5 percent level = 9·49. Since the value of χ^2, calculated as 2·59 is less than the tabulated value, the fit is taken as a good one.

8. HYPERGEOMETRIC MODEL

8.1 In the binomial model, the probability of getting a defective piece p is assumed to be constant. The model is still valid even if p is fairly constant. Thus while selecting 100 items from a lot of say 800 items, the probability from one selection to the next is only approximately constant unless the sampling with replacement is done, that is, each item after selection is put back in the lot before the next item is selected. If there are 16 defectives in the lot then the probability of a good item on the first selection is 784/800 = 0·98. If the first one is good, the probability of a good item on the second drawing is 783/799 or if the first item is defective, the probability of getting a good item in the second draw is 784/799. Both of them are quite close to 0·98. Thus working on the assumption of binomial model is fairly justified. In fact binomial model is truly correct when the lot size is infinite but it still gives useful approximations when the lot size is 8 to 10 times larger than the sample size. But when the lot size is small, it is more justified to use the hypergeometric model which states that if there are D defectives in a lot of N items then the probability of getting exactly r defectives in a sample of size n is given by:

$$P(r) = \frac{\binom{N-D}{n-r}\binom{D}{r}}{\binom{N}{n}}$$

8.1.1 Actually hypergeometric model is the case of sampling from finite population whereas binomial is the case of sampling from infinite population. Hypergeometric models are useful in the calculation of exact probabilities when lot sizes are small and the sample size forms a substantial proportion of the lot size. It is also needed when sampling is done without replacement. Direct calculation of the above probability is very laborious. If the lot size does not exceed 100, the tables of logarithms of factorials given in Appendix E may be utilized.

8.1.2 *Example* — In a lot of size 80 with 7 defectives a sample of size 20 is drawn. Calculate the probability that the sample contains

a) no defective,

b) one defective, and

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*See Appendix A of IS : 6200 (Part II)-1977 ' Statistical test of significance: Part II χ^2 — test(*first revision*). '

c) two defectives.
Here $N = 80$, $D = 7$, $n = 20$

Thus $P(0)$ (the probability of getting 0 defectives) is given by:

$$P(0) = \frac{\binom{73}{20}\binom{7}{0}}{\binom{80}{20}} = \frac{73\,!\;\;20\,!\;\;60\,!}{20\,!\;\;53\,!\;\;80\,!}$$

$$\log P(0) = -\,0{\cdot}915\,1$$

$$P(0) = 0{\cdot}1216$$

$$\text{Similarly } P(1) = \frac{\binom{73}{19}\binom{7}{1}}{\binom{80}{20}} = 0{\cdot}315\,2$$

$$\text{and } P(2) = \frac{\binom{73}{18}\binom{7}{2}}{\binom{80}{20}} = 0{\cdot}326\,7$$

8.1.3 A shipment of 100 television picture tubes contains five defectives. If three tubes are randomly selected and given to a customer, find the probability that he will get (a) exactly one defective tube, (b) two defective tubes, and (c) all the three defective tubes.

Here $N = 100$, $n = 3$ and $D = 5$.

The probability of exactly one defective is:

$$P(1) = \frac{\binom{95}{2}\binom{5}{1}}{\binom{100}{3}}$$

$$= 0{\cdot}138\,07$$

Similarly probability of getting exactly 2 defective tubes is:

$$P(2) = \frac{\binom{95}{1}\binom{5}{2}}{\binom{100}{3}} = 0{\cdot}005\,88$$

and the probability of getting all the 3 defective tubes is:

$$P(3) = \frac{\binom{95}{0}\binom{5}{3}}{\binom{100}{3}} = 0{\cdot}000\ 06$$

8.2 Mean and Variance

$$\text{Mean } E(r) = \frac{n}{N} D = np \text{ where } p = \frac{D}{N} \text{ and } q = \left(1 - \frac{D}{N}\right)$$

$$\text{Variance } V(r) = \frac{nD}{N}\left(1 - \frac{D}{N}\right)\frac{N-n}{N-1} = npq\ \frac{N-n}{N-1}$$

$$\text{and Standard deviation } = \sqrt{V(r)} = \left[n \frac{nD}{N}\left(1 - \frac{D}{N}\right) \frac{N-n}{N-1}\right]^{1/2}$$

$$= \sqrt{npq\ \frac{N-n}{N-1}}$$

8.2.1 *Example* — From a lot of 200 components containing 16 defectives, a sample of size 25 is chosen. Find the expected number of defectives in the sample as also the standard deviation.

$$\text{Expected No. of defectives} = \frac{25}{200} \times 16 = 2$$

$$\text{Standard deviation} = \sqrt{25\left(\frac{18}{200}\right)\left(1 - \frac{16}{200}\right) \times \frac{200-25}{200-1}}$$

$$= \sqrt{\frac{25 \times 16 \times 184 \times 175}{200 \times 200 \times 199}}$$

$$= \sqrt{1{\cdot}618\ 1}$$

$$= 1{\cdot}27$$

APPENDIX A

(*Clause* 5.2.1)

BINOMIAL DISTRIBUTION INDIVIDUAL TERMS

For $n = 2$ to 20 and $p = 0.01 (0.01) .10, (0.05) .50$

$$\binom{n}{r} p^r (1-p)^{n-r}$$

n	r ↓ \ p →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.0198	0.0392	0.0582	0.0768	0.0950	0.1128	0.1302	0.1472	0.1638	0.1800	0.2550	0.3200	0.3750	0.4200	0.4550	0.4800	0.4950	0.5000
	2	0.0001	0.0004	0.0009	0.0016	0.0025	0.0036	0.0049	0.0064	0.0081	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	0.2500
3	0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.0294	0.0576	0.0847	0.1106	0.1354	0.1590	0.1816	0.2031	0.2236	0.2430	0.3251	0.3840	0.4219	0.4410	0.4436	0.4320	0.4084	0.3750
	2	0.0003	0.0012	0.0026	0.0046	0.0071	0.0102	0.0137	0.0177	0.0221	0.0270	0.0574	0.0960	0.1406	0.1890	0.2389	0.2880	0.3341	0.3750
	3				0.0001	0.0002	0.0002	0.0003	0.0005	0.0007	0.0010	0.0034	0.0080	0.0156	0.0270	0.0429	0.0640	0.0911	0.1250
4	0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.0388	0.0753	0.1095	0.1416	0.1715	0.1993	0.2252	0.2492	0.2713	0.2916	0.3685	0.4096	0.4219	0.4116	0.3845	0.3456	0.2995	0.2500
	2	0.0006	0.0023	0.0051	0.0088	0.0135	0.0191	0.0254	0.0325	0.0402	0.0486	0.0975	0.1536	0.2109	0.2646	0.3105	0.3456	0.3675	0.3750
	3			0.0001	0.0002	0.0005	0.0008	0.0013	0.0019	0.0027	0.0036	0.0115	0.0256	0.0469	0.0726	0.1115	0.1536	0.2005	0.2500
	4									0.0001	0.0001	0.0005	0.0016	0.0039	0.0081	0.0150	0.0256	0.0410	0.0625
5	0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
	1	0.0480	0.0922	0.1328	0.1699	0.2036	0.2342	0.2618	0.2866	0.3086	0.3280	0.3915	0.4096	0.3955	0.3602	0.3124	0.2592	0.2059	0.1562
	2	0.0010	0.0038	0.0082	0.0142	0.0214	0.0299	0.0394	0.0498	0.0610	0.0729	0.1382	0.2048	0.2637	0.3087	0.3364	0.3456	0.3369	0.3125
	3		0.0001	0.0003	0.0006	0.0011	0.0019	0.0030	0.0043	0.0060	0.0081	0.0244	0.0512	0.0879	0.1323	0.1811	0.2304	0.2757	0.3125
	4						0.0001	0.0001	0.0002	0.0003	0.0004	0.0022	0.0064	0.0146	0.0284	0.0488	0.0768	0.1128	0.1562
	5											0.0001	0.0003	0.0010	0.0024	0.0053	0.0102	0.0185	0.0312
6	0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.0571	0.1085	0.1546	0.1957	0.2321	0.2642	0.2922	0.3164	0.3370	0.3543	0.3993	0.3932	0.3560	0.3025	0.2437	0.1866	0.1359	0.0938
	2	0.0014	0.0055	0.0120	0.0204	0.0305	0.0422	0.0550	0.0688	0.0833	0.0984	0.1762	0.2458	0.2966	0.3241	0.3280	0.3110	0.2780	0.2344
	3		0.0002	0.0005	0.0011	0.0021	0.0036	0.0055	0.0080	0.0110	0.0116	0.0415	0.0819	0.1318	0.1852	0.2355	0.2765	0.3032	0.3125
	4					0.0001	0.0002	0.0003	0.0005	0.0008	0.0012	0.0055	0.0154	0.0330	0.0595	0.0951	0.1382	0.1861	0.2344
	5										0.0001	0.0004	0.0015	0.0044	0.0102	0.0205	0.0369	0.0609	0.0938
	6												0.0001	0.0002	0.0007	0.0018	0.0041	0.0083	0.0156

(*Continued*)

BINOMIAL DISTRIBUTION INDIVIDUAL TERMS – *Contd*

n	r ↓ \ p →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
7	0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.0659	0.1240	0.1749	0.2192	0.2573	0.2897	0.3170	0.3396	0.3578	0.3720	0.3960	0.3670	0.3115	0.2471	0.1848	0.1306	0.0872	0.0547
	2	0.0020	0.0076	0.0162	0.0274	0.0406	0.0555	0.0716	0.0886	0.1061	0.1240	0.2097	0.2753	0.3115	0.3177	0.2985	0.2613	0.2140	0.1641
	3		0.0003	0.0008	0.0019	0.0036	0.0059	0.0090	0.0128	0.0175	0.0230	0.0617	0.1147	0.1730	0.2269	0.2679	0.2903	0.2918	0.2734
	4				0.0001	0.0002	0.0004	0.0007	0.0011	0.0017	0.0026	0.0109	0.0287	0.0577	0.0972	0.1442	0.1935	0.2388	0.2734
	5								0.0001	0.0001	0.0002	0.0012	0.0043	0.0115	0.0250	0.0466	0.0774	0.1172	0.1641
	6											0.0001	0.0004	0.0013	0.0036	0.0084	0.0172	0.0320	0.0547
	7													0.0001	0.0002	0.0006	0.0016	0.0037	0.0078
8	0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.0746	0.1389	0.1939	0.2405	0.2793	0.3113	0.3370	0.3570	0.3721	0.3826	0.3847	0.3355	0.2670	0.1977	0.1373	0.0896	0.0548	0.0312
	2	0.0026	0.0099	0.0210	0.0351	0.0515	0.0695	0.0888	0.1087	0.1288	0.1488	0.2376	0.2936	0.3115	0.2965	0.2587	0.2090	0.1569	0.1094
	3	0.0001	0.0004	0.0013	0.0029	0.0054	0.0089	0.0134	0.0189	0.0255	0.0331	0.0839	0.1468	0.2076	0.2541	0.2786	0.2787	0.2568	0.2188
	4			0.0001	0.0002	0.0004	0.0007	0.0013	0.0021	0.0031	0.0046	0.0185	0.0459	0.0865	0.1361	0.1875	0.2322	0.2627	0.2734
	5							0.0001	0.0001	0.0002	0.0004	0.0026	0.0092	0.0231	0.0467	0.0808	0.1239	0.1719	0.2188
	6											0.0002	0.0011	0.0038	0.0100	0.0217	0.0413	0.0703	0.1094
	7												0.0001	0.0004	0.0012	0.0033	0.0079	0.0164	0.0312
	8														0.0001	0.0002	0.0007	0.0017	0.0039
9	0	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.0830	0.1531	0.2116	0.2597	0.2985	0.3292	0.3525	0.3695	0.3809	0.3874	0.3679	0.3020	0.2253	0.1556	0.1004	0.0605	0.0339	0.0176
	2	0.0034	0.0125	0.0262	0.0433	0.0629	0.0840	0.1061	0.1285	0.1507	0.1722	0.2597	0.3020	0.3003	0.[illegible]668	0.2162	0.1612	0.1110	0.0703
	3	0.0001	0.0006	0.0019	0.0042	0.0077	0.0125	0.0186	0.0261	0.0348	0.0446	0.1069	0.1762	0.2336	0.2688	0.2716	0.2508	0.2119	0.1641
	4			0.0001	0.0003	0.0006	0.0012	0.0021	0.0034	0.0052	0.0074	0.0283	0.0661	0.1168	0.1715	0.2194	0.2508	0.2600	0.2461
	5						0.0001	0.0002	0.0003	0.0005	0.0008	0.0050	0.0165	0.0389	0.0735	0.1181	0.1672	0.2128	0.2461
	6										0.0001	0.0006	0.0028	0.0087	0.0210	0.0424	0.0743	0.1160	0.1641
	7												0.0003	0.0012	0.0039	0.0098	0.0212	0.0407	0.0703
	8													0.0001	0.0004	0.0013	0.0035	0.0083	0.0176
	9															0.0001	0.0003	0.0008	0.0020
10	0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.0914	0.1667	0.2281	0.2770	0.3151	0.3438	0.3643	0.3777	0.3851	0.3874	0.3474	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098
	2	0.0042	0.0153	0.0317	0.0519	0.0746	0.0988	0.1234	0.1478	0.1714	0.1937	0.2759	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439
	3	0.0001	0.0008	0.0026	0.0058	0.0105	0.0168	0.0248	0.0343	0.0452	0.0574	0.1298	0.2013	0.2503	0.2668	0.2522	0.2150	0.1665	0.1172
	4			0.0001	0.0004	0.0010	0.0019	0.0033	0.0052	0.0078	0.0112	0.0401	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051
	5					0.0001	0.0001	0.0003	0.0005	0.0009	0.0015	0.0085	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461
	6									0.0001	0.0001	0.0012	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051
	7											0.0001	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172
	8												0.0001	0.0004	0.0014	0.0043	0.0106	0.0229	0.0439
	9														0.0001	0.0005	0.0016	0.0042	0.0098
	10																0.0001	0.0003	0.0010

(*Continued*)

BINOMIAL DISTRIBUTION : INDIVIDUAL TERMS – *Contd*

n	r ↓ p →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
11	0	0.8953	0.8007	0.7153	0.6382	0.5688	0.5063	0.4501	0.3996	0.3544	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.0995	0.1798	0.2433	0.2925	0.3293	0.3555	0.3727	0.3823	0.3855	0.3835	0.3248	0.2362	0.1549	0.0932	0.0518	0.0266	0.0125	0.0054
	2	0.0050	0.0183	0.0376	0.0609	0.0867	0.1135	0.1403	0.1662	0.1906	0.2131	0.2866	0.2953	0.2581	0.1998	0.1395	0.0887	0.0513	0.0269
	3	0.0002	0.0011	0.0035	0.0076	0.0137	0.0217	0.0317	0.0434	0.0566	0.0710	0.1517	0.2215	0.2581	0.2568	0.2254	0.1774	0.1259	0.0806
	4			0.0002	0.0006	0.0014	0.0028	0.0048	0.0075	0.0112	0.0158	0.0536	0.1107	0.0721	0.2201	0.2428	0.2365	0.2060	0.1611
	5					0.0001	0.0002	0.0005	0.0009	0.0015	0.0025	0.0132	0.0388	0.0803	0.1321	0.1830	0.2207	0.2360	0.2256
	6								0.0001	0.0002	0.0003	0.0023	0.0097	0.0268	0.0566	0.0985	0.1471	0.1931	0.2256
	7											0.0003	0.0017	0.0064	0.0173	0.0379	0.0701	0.1128	0.1611
	8												0.0002	0.0011	0.0037	0.0102	0.0234	0.0462	0.0806
	9													0.0001	0.0005	0.0018	0.0052	0.0126	0.0269
	10															0.0002	0.0007	0.0021	0.0054
	11																	0.0002	0.0005
12	0	0.8864	0.7847	0.6938	0.6127	0.5404	0.4759	0.4186	0.3677	0.3225	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
	1	0.1074	0.1922	0.2575	0.3064	0.3413	0.3645	0.3781	0.3837	0.3827	0.3766	0.3012	0.2062	0.1267	0.0712	0.0368	0.0174	0.0075	0.0029
	2	0.0060	0.0216	0.0438	0.0702	0.0988	0.1280	0.1565	0.1835	0.2082	0.2301	0.2924	0.2835	0.2323	0.1678	0.1088	0.0639	0.0339	0.0161
	3	0.0002	0.0015	0.0045	0.0098	0.0173	0.0272	0·0393	0.0532	0.0686	0.0852	0.1720	0.2362	0.2581	0.2397	0.1954	0.1419	0.0923	0.0537
	4		0.0001	0.0003	0.0009	0.0021	0.0039	0.0067	0.0104	0.0153	0.0213	0.0683	0.1329	0.1936	0.2311	0.2367	0.2128	0.1700	0.1208
	5				0.0001	0.0002	0.0004	0.0008	0.0014	0.0024	0.0038	0.0193	0.0532	0.1032	0.1585	0.2039	0.2270	0.2225	0.1934
	6							0.0001	0.0001	0.0003	0.0005	0.0040	0.0155	0.0401	0.0792	0.1281	0.1766	0.2124	0.2256
	7										0.0001	0.0006	0.0033	0.0115	0.0291	0.0591	0.1009	0.1489	0.1934
	8											0.0001	0.0005	0.0024	0.0078	0.0199	0.0420	0.0762	0.1208
	9												0.0001	0.0004	0.0015	0.0048	0.0125	0.0277	0.0537
	10														0.0002	0.0008	0.0025	0.0068	0.0161
	11															0.0001	0.0003	0.0010	0.0029
	12																	0.0001	0.0002
13	0	0.8775	0.7690	0.6730	0.5882	0.5133	0.4474	0.3893	0.3383	0.2935	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001
	1	0.1152	0.2040	0.2706	0.3186	0.3512	0.3712	0.3809	0.3824	0.3773	0.3672	0.2774	0.1787	0.1029	0.0540	0.0259	0.0113	0.0045	0.0016
	2	0.0070	0.0250	0.0502	0.0797	0.1109	0.1422	0.1720	0.1995	0.2239	0.2448	0.2937	0.2680	0.2059	0.1388	0.0836	0.0453	0.0220	0.0095
	3	0.0003	0.0019	0.0057	0.0122	0.0214	0.0333	0.0475	0.0636	0.0812	0.0997	0.1900	0.2457	0.2517	0.2181	0.1651	0.1107	0.0660	0.0349
	4		0.0001	0.0004	0.0013	0.0028	0.0053	0.0089	0.0138	0.0201	0.0277	0.0838	0.1535	0.2097	0.2337	0.2222	0.1845	0.1350	0.0872
	5				0.0001	0.0003	0.0006	0.0012	0.0022	0.0036	0.0055	0.0266	0.0691	0.1258	0.1803	0.2154	0.2214	0.1989	0.1571
	6						0.0001	0.0001	0.0003	0.0005	0.0008	0.0063	0.0230	0.0559	0.1030	0.1546	0.1968	0.2169	0.2095
	7										0.0001	0.0011	0.0058	0.0186	0.0449	0.0833	0.1312	0.1775	0.2095
	8											0.0001	0.0011	0.0047	0.0142	0.0336	0.0056	0.1089	0.1571
	9												0.0001	0.0009	0.0034	0.0101	0.0243	0.0495	0.0873
	10													0.0001	0.0006	0.0022	0.0065	0.0162	0.0349
	11														0.0001	0.0003	0.0012	0.0036	0.0095
	12																0.0001	0.0005	0.0016
	13																		0.0001
14	0	0.8687	0.7536	0.6528	0.5647	0.4877	0.4205	0.3620	0.3112	0.2670	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001
	1	0.1229	0.2153	0.2827	0.3294	0.3593	0.3758	0.3815	0.3788	0.3698	0.3559	0.2539	0.1539	0.0832	0.0407	0.0181	0.0073	0.0027	0.0009
	2	0.0081	0.0286	0.0568	0.0892	0.1229	0.1559	0.1867	0.2141	0.2377	0.2570	0.2912	0.2501	0.1802	0.1134	0.0634	0.0317	0.0141	0.0056
	3	0.0003	0.0023	0.0070	0.0149	0.0259	0.0398	0.0562	0.0745	0.0940	0.1142	0.2056	0.2501	0.2402	0.1943	0.1366	0.0845	0.0462	0.0222
	4		0.0001	0.0006	0.0017	0.0037	0.0070	0.0116	0.0178	0.0256	0.0349	0.0998	0.1720	0.2202	0.2290	0.2022	0.1549	0.1040	0.0611
	5				0.0001	0.0004	0.0009	0.0018	0.0031	0.0051	0.0078	0.0352	0.0860	0.1468	0.1963	0.2178	0.2066	0.1701	0.1222
	6						0.0001	0.0002	0.0004	0.0008	0.0013	0.0093	0.0322	0.0734	0.1262	0.1759	0.2066	0.2088	0.1833
	7									0.0001	0.0002	0.0019	0.0092	0.0280	0.0618	0.1082	0.1574	0.1952	0.2095
	8											0.0003	0.0020	0.0082	0.0232	0.0510	0.0918	0.1398	0.1833
	9												0.0003	0.0018	0.0066	0.0183	0.0408	0.0762	0.1222
	10													0.0003	0.0014	0.0049	0.0136	0.0312	0.0611
	11														0.0002	0.0010	0.0033	0.0093	0.0222
	12															0.0001	0.0005	0.0019	0.0056
	13																0.0001	0.0002	0.0009
	14																		0.0001

(*Continued*)

BINOMIAL DISTRIBUTION : INDIVIDUAL TERMS – *Contd*

n	r ↓ \ p →	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
15	0	0·8601	0·7386	0·6333	0·5421	0·4633	0·3953	0·3367	0·2863	0·2430	0·2059	0·0874	0·0352	0·0134	0·0047	0·0016	0·0005	0·0001	—
	1	0·1303	0·2761	0·2938	0·3388	0·3658	0·3785	0·3801	0·3734	0·3605	0·3432	0·2312	0·1319	0·0668	0·0305	0·0126	0·0047	0·0016	0·0005
	2	0·0092	0·0323	0·0636	0·0988	0·1348	0·1691	0·2003	0·2273	0·2496	0·2669	0·2856	0·2309	0·1559	0·0916	0·0476	0·0219	0·0090	0·0032
	3	0·0004	0·0029	0·0085	0·0178	0·0307	0·0468	0·0653	0·0857	0·1070	0·1285	0·2184	0·2501	0·2252	0·1700	0·1110	0·0634	0·0318	0·0139
	4		0·0002	0·0008	0·0022	0·0049	0·0090	0·0148	0·0223	0·0317	0·0428	0·1156	0·1876	0·2252	0·2186	0·1792	0·1268	0·0780	0·0417
	5			0·0001	0·0002	0·0006	0·0013	0·0024	0·0043	0·0069	0·0105	0·0449	0·1032	0·1651	0·2061	0·2123	0·1859	0·1404	0·0916
	6						0·0001	0·0003	0·0006	0·0011	0·0019	0·0132	0·0430	0·0917	0·1472	0·1906	0·2066	0·1914	0·1527
	7								0·0001	0·0001	0·0003	0·0030	0·0138	0·0393	0·0811	0·1319	0·1771	0·2013	0·1964
	8											0·0005	0·0035	0·0131	0·0348	0·0710	0·1181	0·1647	0·1964
	9											0·0001	0·0007	0·0034	0·0116	0·0298	0·0612	0·1048	0·1527
	10												0·0001	0·0007	0·0030	0·0096	0·0245	0·0515	0·0916
	11													0·0001	0·0006	0·0024	0·0074	0·0191	0·0417
	12														0·0001	0·0004	0·0016	0·0052	0·0139
	13															0·0001	0·0003	0·0010	0·0032
	14																	0·0001	0·0005
	15																		
16	0	0·8515	0·7236	0·6143	0·5204	0·4401	0·3716	0·3131	0·2634	0·2211	0·1853	0·0743	0·0281	0·0100	0·0033	0·0010	0·0003	0·0001	—
	1	0·1376	0·2363	0·3040	0·3469	0·3706	0·3795	0·3771	0·3665	0·3499	0·3294	0·2097	0·1126	0·0535	0·0228	0·0087	0·0030	0·0009	0·0002
	2	0·0104	0·0362	0·0705	0·1034	0·1463	0·1817	0·2129	0·2390	0·2596	0·2745	0·2775	0·2111	0·1336	0·0732	0·0353	0·0150	0·0056	0·0018
	3	0·0005	0·0034	0·0102	0·0211	0·0359	0·0541	0·0748	0·0970	0·1198	0·1423	0·2285	0·2463	0·2079	0·1465	0·0888	0·0468	0·0215	0·0085
	4		0·0002	0·0010	0·0029	0·0061	0·0112	0·0183	0·0274	0·0385	0·0514	0·1311	0·2001	0·2252	0·2040	0·1553	0·1014	0·0572	0·0278
	5			0·0001	0·0003	0·0008	0·0017	0·0033	0·0057	0·0091	0·0137	0·0555	0·1201	0·1802	0·2099	0·2008	0·1623	0·1123	0·0667
	6					0·0001	0·0002	0·0005	0·0009	0·0017	0·0028	0·0180	0·0550	0·1101	0·1649	0·1982	0·1983	0·1684	0·1222
	7								0·0001	0·0002	0·0004	0·0045	0·0197	0·0524	0·1010	0·1524	0·1889	0·1969	0·1746
	8										0·0001	0·0009	0·0055	0·0197	0·0487	0·0923	0·1417	0·1812	0·1964
	9											0·0001	0·0012	0·0058	0·0185	0·0442	0·0840	0·1318	0·1746
	10												0·0002	0·0014	0·0056	0·0167	0·0392	0·0755	0·1222
	11													0·0002	0·0013	0·0049	0·0142	0·0337	0·0667
	12														0·0002	0·0011	0·0040	0·0115	0·0278
	13															0·0002	0·0008	0·0029	0·0085
	14																0·0001	0·0005	0·0018
	15																	0·0001	0·0002
	16																		—
17	0	0·8429	0·7093	0·5958	0·4996	0·4181	0·3493	0·2912	0·2423	0·2012	0·1668	0·0631	0·0225	0·0075	0·0023	0·0007	0·0002	—	—
	1	0·1447	0·2461	0·3133	0·3539	0·3741	0·3790	0·3726	0·3582	0·3383	0·3150	0·1893	0·0957	0·0426	0·0169	0·0060	0·0019	0·0005	0·0001
	2	0·0117	0·0402	0·0775	0·1180	0·1575	0·1935	0·2244	0·2492	0·2677	0·2800	0·2673	0·1914	0·1136	0·0581	0·0260	0·0102	0·0035	0·0010
	3	0·0006	0·0041	0·0120	0·0246	0·0415	0·0618	0·0844	0·1083	0·1324	0·1556	0·2359	0·2593	0·1893	0·1245	0·0701	0·0341	0·0144	0·0052
	4		0·0003	0·0013	0·0036	0·0076	0·0138	0·0222	0·0330	0·0458	0·0605	0·1457	0·2093	0·2209	0·1868	0·1320	0·0796	0·0411	0·0182
	5			0·0001	0·0004	0·0010	0·0023	0·0044	0·0075	0·0118	0·0175	0·0668	0·1361	0·1914	0·2081	0·1849	0·1379	0·0875	0·0472
	6					0·0001	0·0003	0·0007	0·0013	0·0023	0·0039	0·0236	0·0680	0·1276	0·1784	0·1991	0·1839	0·1432	0·0944
	7							0·0001	0·0002	0·0004	0·0007	0·0065	0·0267	0·0668	0·1201	0·1685	0·1927	0·1841	0·1484
	8										0·0001	0·0014	0·0084	0·0279	0·0644	0·1134	0·1606	0·1883	0·1855
	9											0·0003	0·0021	0·0093	0·0276	0·0611	0·1070	0·1540	0·1855
	10												0·0004	0·0025	0·0095	0·0263	0·0571	0·1008	0·1484
	11												0·0001	0·0005	0·0026	0·0090	0·0242	0·0525	0·0944
	12													0·0001	0·0006	0·0024	0·0081	0·0215	0·0472
	13														0·0001	0·0005	0·0021	0·0068	0·0182
	14															0·0001	0·0004	0·0016	0·0052
	15																0·0001	0·0003	0·0010
	16																		0·0001
	17																		—

(*Continued*)

BINOMIAL DISTRIBUTION : INDIVIDUAL TERMS — *Contd*

n	r ↓ p →	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
18	0	0·8345	0·6951	0·5780	0·4796	0·3972	0·3283	0·2708	0·2229	0·1831	0·1501	0·0536	0·0180	0·0056	0·0016	0·0004	0·0001	—	—
	1	0·1517	0·2554	0·3217	0·3597	0·3763	0·3772	0·3669	0·3489	0·3260	0·3002	0·1704	0·0811	0·0338	0·0126	0·0042	0·0012	0·0003	0·0001
	2	0·0130	0·0443	0·0846	0·1274	0·1683	0·2047	0·2348	0·2579	0·2741	0·2835	0·2556	0·1723	0·0958	0·0458	0·0190	0·0069	0·0022	0·0006
	3	0·0007	0·0048	0·0140	0·0283	0·0473	0·0697	0·0942	0·1196	0·1446	0·1680	0·2400	0·2297	0·1704	0·1046	0·0547	0·0246	0·0095	0·0031
	4		0·0004	0·0016	0·0044	0·0093	0·0167	0·0266	0·0390	0·0536	0·0700	0·1592	0·2153	0·2130	0·1681	0·1104	0·0614	0·0291	0·0117
	5			0·0001	0·0005	0·0014	0·0030	0·0056	0·0095	0·0148	0·0218	0·0787	0·1507	0·1988	0·2017	0·1664	0·1146	0·0666	0·0327
	6					0·0002	0·0004	0·0009	0·0018	0·0032	0·0052	0·0301	0·0816	0·1436	0·1873	0·1941	0·1655	0·1181	0·0708
	7							0·0001	0·0003	0·0005	0·0010	0·0091	0·0350	0·0820	0·1376	0·1792	0·1892	0·1657	0·1214
	8									0·0001	0·0002	0·0022	0·0120	0·0376	0·0811	0·1327	0·1734	0·1864	0·1669
	9											0·0004	0·0033	0·0139	0·0386	0·0794	0·1284	0·1694	0·1855
	10											0·0001	0·0008	0·0042	0·0149	0·0385	0·0771	0·1248	0·1669
	11												0·0001	0·0010	0·0046	0·0151	0·0374	0·0742	0·1214
	12													0·0002	0·0012	0·0047	0·0145	0·0354	0·0708
	13														0·0002	0·0012	0·0045	0·0134	0·0327
	14															0·0002	0·0011	0·0039	0·0117
	15																0·0002	0·0009	0·0031
	16																	0·0001	0·0006
	17																		0·0001
	18																		—
19	0	0·8262	0·6812	0·5606	0·4604	0·3774	0·3086	0·2519	0·2051	0·1666	0·1351	0·0456	0·0144	0·0042	0·0011	0·0003	0·0001	—	—
	1	0·1586	0·2642	0·3294	0·3645	0·3774	0·3743	0·3602	0·3389	0·3131	0·2852	0·1529	0·0685	0·0268	0·0093	0·0029	0·0008	0·0002	—
	2	0·0144	0·0485	0·0917	0·1367	0·1787	0·2150	0·2440	0·2652	0·2787	0·2852	0·2428	0·1540	0·0803	0·0358	0·0138	0·0046	0·0013	0·0003
	3	0·0008	0·0056	0·0161	0·0323	0·0533	0·0778	0·1041	0·1307	0·1562	0·1796	0·2428	0·2182	0·1517	0·0869	0·0422	0·0175	0·0062	0·0018
	4		0·0005	0·0020	0·0054	0·0112	0·0199	0·0313	0·0455	0·0618	0·0798	0·1714	0·2182	0·2023	0·1491	0·0909	0·0467	0·0203	0·0074
	5			0·0002	0·0007	0·0018	0·0038	0·0071	0·0119	0·0183	0·0266	0·0907	0·1636	0·2023	0·1916	0·1468	0·0933	0·0497	0·0222
	6				0·0001	0·0002	0·0006	0·0012	0·0024	0·0042	0·0069	0·0374	0·0955	0·1574	0·1916	0·1844	0·1451	0·0949	0·0518
	7						0·0001	0·0002	0·0004	0·0008	0·0014	0·0122	0·0443	0·0974	0·1525	0·1844	0·1797	0·1443	0·0961
	8								0·0001	0·0001	0·0002	0·0032	0·0166	0·0487	0·0981	0·1489	0·1797	0·1771	0·1442
	9											0·0007	0·0051	0·0198	0·0514	0·0980	0·1464	0·1771	0·1762
	10											0·0001	0·0013	0·0066	0·0220	0·0528	0·0976	0·1449	0·1762
	11												0·0003	0·0018	0·0077	0·0233	0·0532	0·0970	0·1442
	12													0·0004	0·0022	0·0083	0·0237	0·0529	0·0961
	13													0·0001	0·0005	0·0024	0·0085	0·0233	0·0518
	14														0·0001	0·0006	0·0024	0·0082	0·0222
	15															0·0001	0·0005	0·0022	0·0074
	16																0·0001	0·0005	0·0018
	17																	0·0001	0·0003
	18																		—
	19																		—
20	0	0·8179	0·6676	0·5438	0·4420	0·3585	0·2901	0·2342	0·1887	0·1516	0·1216	0·0388	0·0115	0·0032	0·0008	0·0002	—	—	—
	1	0·1652	0·2725	0·3364	0·3683	0·3774	0·3703	0·3526	0·3282	0·3000	0·2702	0·1368	0·0576	0·0211	0·0068	0·0020	0·0005	0·0001	—
	2	0·0159	0·0528	0·0988	0·1458	0·1887	0·2246	0·2521	0·2711	0·2818	0·2852	0·2293	0·1369	0·0669	0·0278	0·0100	0·0031	0·0008	0·0002
	3	0·0010	0·0065	0·0183	0·0364	0·0596	0·0860	0·1139	0·1414	0·1672	0·1901	0·2428	0·2054	0·1339	0·0716	0·0323	0·0123	0·0040	0·0011
	4		0·0006	0·0024	0·0065	0·0133	0·0233	0·0364	0·0523	0·0703	0·0898	0·1821	0·2182	0·1897	0·1304	0·0738	0·0350	0·0139	0·0046
	5		—	0·0002	0·0009	0·0022	0·0048	0·0088	0·0145	0·0222	0·0319	0·1028	0·1746	0·2023	0·1789	0·1272	0·0746	0·0365	0·0148
	6				0·0001	0·0003	0·0008	0·0017	0·0032	0·0055	0·0089	0·0454	0·1091	0·1686	0·1916	0·1712	0·1244	0·0746	0·0370
	7						0·0001	0·0002	0·0005	0·0011	0·0020	0·0160	0·0545	0·1124	0·1643	0·1844	0·1659	0·1221	0·0739
	8								0·0001	0·0002	0·0004	0·0046	0·0222	0·0609	0·1144	0·1614	0·1797	0·1623	0·1201
	9										0·0001	0·0011	0·0074	0·0271	0·0654	0·1158	0·1597	0·1771	0·1602
	10											0·0002	0·0020	0·0099	0·0308	0·0686	0·1171	0·1593	0·1762
	11												0·0005	0·0030	0·0120	0·0336	0·0710	0·1185	0·1602
	12												0·0001	0·0008	0·0039	0·0136	0·0355	0·0727	0·1201
	13													0·0002	0·0010	0·0045	0·0146	0·0366	0·0739
	14														0·0002	0·0012	0·0049	0·0150	0·0370
	15															0·0003	0·0013	0·0049	0·0148
	16																0·0003	0·0013	0·0046
	17																	0·0002	0·0011
	18																	—	0·0002
	19																		—
	20																		—

(*Continued*)

APPENDIX B

(*Clauses* 5.2.1 *and* 5.2.3)

BINOMIAL DISTRIBUTION : PARTIAL SUMS

For $n = 2$ to 20 and $p = 0.01(0.01)0.10(0.05)0.50$

$$\sum_{x=0}^{r} \binom{n}{x} p^x (1-p)^{n-x}$$

n	$r \downarrow$ \ $p \rightarrow$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9999	0.9996	0.9991	0.9984	0.9975	0.9964	0.9951	0.9936	0.9919	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8100	0.7975	0.7500
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.3120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9997	0.9988	0.9974	0.9953	0.9928	0.9896	0.9860	0.9818	0.9772	0.9720	0.9392	0.8960	0.8438	0.7840	0.7182	0.6480	0.5748	0.5000
	2	1.0000	1.0000	0.1000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
	3				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.9994	0.9977	0.9948	0.9909	0.9860	0.9801	0.9733	0.9656	0.9570	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9992	0.9987	0.9981	0.9973	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875
	3			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375
	4									1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
	1	0.9990	0.9962	0.9915	0.9852	0.9774	0.9681	0.9575	0.9456	0.9326	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
	2	1.0000	0.9999	0.9997	0.9994	0.9988	0.9980	0.9969	0.9955	0.9937	0.9914	0.9734	0.9421	0.8935	0.8369	0.7648	0.6826	0.5931	0.5000
	3		1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
	4						1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
	5											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0167	0.0277	0.0156
	1	0.9985	0.9943	0.9875	0.9784	0.9672	0.9541	0.9392	0.9227	0.9048	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	2	1.0000	0.9998	0.9995	0.9988	0.9978	0.9962	0.9942	0.9915	0.9882	0.9842	0.9527	0.8011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3138
	3		1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6562
	4					1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
	5										1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
	6												1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4785	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9980	0.9921	0.9829	0.9706	0.9556	0.9382	0.9187	0.8974	0.8745	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	2	1.0000	0.9997	0.9991	0.9980	0.9962	0.9937	0.9903	0.9860	0.9807	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
	3		1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9982	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	4				1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
	5								1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9613	0.9375
	6											1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
	7													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(*Continued*)

BINOMIAL DISTRIBUTION : PARTIAL SUMS – *Contd*

n	r ↓ p →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
8	0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9973	0.9897	0.9777	0.9619	0.9428	0.9208	0.8965	0.8702	0.8423	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
	2	0.9999	0.9996	0.9987	0.9969	0.9942	0.9904	0.9853	0.9789	0.9711	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	3	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
	4			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	5							1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
	6											1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	7												1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
	8														1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9966	0.9869	0.9718	0.9522	0.9288	0.9022	0.8729	0.8417	0.8088	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9999	0.9994	0.9980	0.9955	0.9916	0.9862	0.9791	0.9702	0.9595	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9977	0.9963	0.9943	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4			1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5						1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6										1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7												1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8													1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
	9															1.0000	1.0000	1.0000	1.0000
10	0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9957	0.9838	0.9655	0.9418	0.9139	0.8824	0.8483	0.8121	0.7746	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
	2	0.9999	0.9991	0.9972	0.9938	0.9885	0.9812	0.9717	0.9599	0.9460	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	3	1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9964	0.9942	0.9912	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	4			1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	5					1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	6									1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	7											1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	8												1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
	9														1.0000	1.0000	0.9999	0.9997	0.9990
	10																1.0000	1.0000	1.0000
11	0	0.8953	0.8007	0.7153	0.6382	0.5688	0.5063	0.4501	0.3996	0.3544	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.9948	0.9805	0.9587	0.9308	0.8981	0.8618	0.8228	0.7819	0.7399	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
	2	0.9998	0.9988	0.9963	0.9917	0.9848	0.9752	0.9630	0.9481	0.9305	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327
	3	1.0000	1.0000	0.9998	0.9993	0.9984	0.9970	0.9947	0.9915	0.9871	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
	4			1.0000	1.0000	0.9999	0.9997	0.9995	0.9990	0.9983	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744
	5					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
	6								1.0000	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256
	7											1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
	8												1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673
	9													1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
	10															1.0000	1.0000	0.9998	0.9995
	11																	1.0000	1.0000

(*Continued*)

BINOMIAL DISTRIBUTION : PARTIAL SUMS – *Contd*

n	p → r ↓	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
12	0	0·8864	0·7847	0·6938	0·6127	0·5404	0·4759	0·4186	0·3677	0·3225	0·2824	0·1422	0·0687	0·0317	0·0138	0·0057	0·0022	0·0008	0·0002
	1	0·9938	0·9769	0·9514	0·9191	0·8816	0·8405	0·7967	0·7513	0·7052	0·6590	0·4435	0·2749	0·1584	0·0850	0·0424	0·0196	0·0083	0·0032
	2	0·9998	0·9985	0·9952	0·9893	0·9804	0·9684	0·9532	0·9348	0·9134	0·8891	0·7358	0·5583	0·3907	0·2528	0·1513	0·0834	0·0421	0·0193
	3	1·0000	0·9999	0·9997	0·9990	0·9978	0·9957	0·9925	0·9880	0·9820	0·9744	0·9078	0·7946	0·6488	0·4925	0·3467	0·2253	0·1345	0·0730
	4		1·0000	1·0000	0·9999	0·9998	0·9996	0·9991	0·9984	0·9973	0·9957	0·9761	0·9274	0·8424	0·7237	0·5833	0·4382	0·3044	0·1938
	5				1·0000	1·0000	1·0000	0·9999	0·9998	0·9997	0·9995	0·9954	0·9806	0·9456	0·8822	0·7873	0·6652	0·5269	0·3872
	6							1·0000	1·0000	1·0000	0·9999	0·9993	0·9961	0·9857	0·9614	0·9154	0·8418	0·7393	0·6128
	7										1·0000	0·9999	0·9994	0·9972	0·9905	0·9745	0·9427	0·8883	0·8062
	8											1·0000	0·9999	0·9996	0·9983	0·9944	0·9847	0·9644	0·9270
	9												1·0000	1·0000	0·9998	0·9992	0·9972	0·9921	0·9807
	10														1·0000	0·9999	0·9997	0·9989	0·9968
	11															1·0000	1·0000	0·9999	0·9998
	12																	1·0000	1·0000
13	0	0·8775	0·7690	0·6730	0·5882	0·5133	0·4474	0·3893	0·3383	0·2935	0·2542	0·1209	0·0550	0·0238	0·0097	0·0037	0·0013	0·0004	0·0001
	1	0·9928	0·9730	0·9436	0·9068	0·8646	0·8186	0·7702	0·7206	0·6707	0·6213	0·3983	0·2336	0·1267	0·0637	0·0296	0·0126	0·0049	0·0017
	2	0·9997	0·9980	0·9938	0·9865	0·9755	0·9608	0·9422	0·9201	0·8946	0·8661	0·6920	0·5017	0·3326	0·2025	0·1132	0·0579	0·0269	0·0112
	3	1·0000	0·9999	0·9995	0·9986	0·9969	0·9940	0·9897	0·9837	0·9758	0·9658	0·8820	0·7473	0·5843	0·4206	0·2783	0·1686	0·0929	0·0461
	4		1·0000	1·0000	0·9999	0·9997	0·9993	0·9987	0·9976	0·9959	0·9935	0·9658	0·9009	0·7940	0·6543	0·5005	0·3530	0·2279	0·1334
	5				1·0000	1·0000	0·9999	0·9999	0·9997	0·9995	0·9991	0·9925	0·9700	0·9198	0·8346	0·7159	0·5744	0·4268	0·2905
	6						1·0000	1·0000	1·0000	0·9999	0·9999	0·9987	0·9930	0·9757	0·9376	0·8705	0·7712	0·6437	0·5000
	7									1·0000	1·0000	0·9998	0·9988	0·9944	0·9818	0·9538	0·9023	0·8212	0·7095
	8											1·0000	0·9998	0·9990	0·9960	0·9874	0·9679	0·9302	0·8666
	9												1·0000	0·9999	0·9993	0·9975	0·9922	0·9797	0·9539
	10													1·0000	0·9999	0·9997	0·9987	0·9959	0·9888
	11														1·0000	1·0000	0·9999	0·9995	0·9983
	12																1·0000	1·0000	0·9999
	13																		1·0000
14	0	0·8687	0·7536	0·6528	0·5647	0·4877	0·4205	0·3620	0·3112	0·2670	0·2288	0·1028	0·0440	0·0178	0·0068	0·0024	0·0008	0·0002	0·0001
	1	0·9916	0·9690	0·9355	0·8941	0·8470	0·7963	0·7436	0·6900	0·6368	0·5846	0·3567	0·1979	0·1010	0·0475	0·0205	0·0081	0·0029	0·0009
	2	0·9997	0·9975	0·9923	0·9833	0·9699	0·9522	0·9302	0·9042	0·8745	0·8416	0·6479	0·4481	0·2811	0·1608	0·0839	0·0398	0·0170	0·0065
	3	1·0000	0·9999	0·9994	0·9981	0·9958	0·9920	0·9864	0·9786	0·9685	0·9559	0·8535	0·6982	0·5213	0·3552	0·2205	0·1243	0·0632	0·0287
	4		1·0000	1·0000	0·9998	0·9996	0·9990	0·9980	0·9965	0·9941	0·9908	0·9533	0·8702	0·7415	0·5842	0·4227	0·2793	0·1672	0·0898
	5				1·0000	1·0000	0·9999	0·9998	0·9996	0·9992	0·9985	0·9885	0·9561	0·8883	0·7805	0·6405	0·4859	0·3373	0·2120
	6						1·0000	1·0000	1·0000	0·9999	0·9998	0·9978	0·9884	0·9617	0·9067	0·8164	0·6925	0·5461	0·3953
	7									1·0000	1·0000	0·9997	0·9976	0·9897	0·9685	0·9247	0·8499	0·7414	0·6047
	8											1·0000	0·9996	0·9978	0·9917	0·9757	0·9417	0·8811	0·7880
	9												1·0000	0·9997	0·9983	0·9940	0·9825	0·9574	0·9102
	10													1·0000	0·9998	0·9989	0·9961	0·9886	0·9713
	11														1·0000	0·9999	0·9994	0·9978	0·9935
	12															1·0000	0·9999	0·9997	0·9991
	13																1·0000	1·0000	0·9999
	14																		1·0000

(*Continued*)

BINOMIAL DISTRIBUTION : PARTIAL SUMS – *Contd*

n	p → r ↓	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
15	0	0.8601	0.7386	0.6333	0.5421	0.4633	0.3953	0.3367	0.2863	0.2430	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	—
	1	0.9904	0.9647	0.9270	0.8809	0.8290	0.7738	0.7168	0.6597	0.6035	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
	2	0.9996	0.9970	0.9906	0.9797	0.9638	0.9429	0.9171	0.8870	0.8531	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
	3	1.0000	0.9998	0.9992	0.9976	0.9945	0.9896	0.9825	0.9727	0.9601	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
	4		1.0000	0.9999	0.9998	0.9994	0.9986	0.9972	0.9950	0.9918	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
	5			1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
	6					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
	7								1.0000	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
	8											0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
	9											1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
	10												1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
	11													1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
	12														1.0000	0.9999	0.9997	0.9989	0.9963
	13															1.0000	1.0000	0.9999	0.9995
	14																	1.0000	1.0000
	15																		1.0000
16	0	0.8515	0.7238	0.6143	0.5204	0.4401	0.3716	0.3131	0.2634	0.2211	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	—
	1	0.9891	0.9601	0.9182	0.8673	0.8108	0.7511	0.6902	0.6299	0.5711	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003
	2	0.9995	0.9963	0.9887	0.9758	0.9571	0.9327	0.9031	0.8689	0.8306	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021
	3	1.0000	0.9998	0.9989	0.9968	0.9930	0.9868	0.9779	0.9658	0.9504	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106
	4		1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9889	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384
	5			1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9981	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051
	6					1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272
	7							1.0000	1.0000	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018
	8										1.0000	0.9998	0.9985	0.9925	0.9743	0.9329	0.8577	0.7441	0.5982
	9											1.0000	0.9998	0.9984	0.9929	0.9771	0.9417	0.8759	0.7728
	10												1.0000	0.9997	0.9984	0.9938	0.9809	0.9514	0.8949
	11													1.0000	0.9997	0.9987	0.9951	0.9851	0.9616
	12														1.0000	0.9998	0.9991	0.9965	0.9894
	13															1.0000	0.9999	0.9994	0.9979
	14																1.0000	0.9999	0.9997
	15																	1.0000	1.0000
	16																		—

(*Continued*)

BINOMIAL DISTRIBUTION : PARTIAL SUMS – *Contd*

n	p→ r↓	0 01	0 02	0 03	0 04	0·05	0 06	0 07	0 08	0 09	0 10	0 15	0 20	0 25	0 30	0 35	0 40	0 45	0 50
17	0	0 8429	0 7093	0 5958	0 4996	0 4181	0 3493	0 2912	0 2423	0 2012	0 1668	0 0631	0 0225	0 0075	0 0023	0 0007	0 0002	—	—
	1	0 9877	0 9554	0 9091	0 8535	0 7922	0 7283	0 6638	0 6005	0 5396	0 4818	0 2525	0 1182	0 0501	0 0193	0 0067	0 0021	0 0006	0 0001
	2	0 9994	0 9956	0 9866	0 9714	0 9497	0 9218	0 8882	0 8497	0 8073	0 7618	0 5198	0 3096	0 1637	0 0774	0 0327	0 0123	0 0041	0 0012
	3	1 0000	0 9997	0 9986	0 9960	0 9912	0 9836	0 9727	0 9581	0 9397	0 9174	0 7556	0 5489	0 3530	0 2019	0 1028	0 0464	0 0184	0 0064
	4		1 0000	0 9999	0 9996	0 9988	0 9974	0 9949	0 9911	0 9855	0 9779	0 9013	0 7582	0 5739	0 3887	0 2348	0 1260	0 0596	0 0245
	5			1 0000	1 0000	0 9999	0 9997	0 9993	0 9985	0 9973	0 9953	0 9681	0 8943	0 7653	0 5968	0 4197	0 2639	0 1471	0 0717
	6					1 0000	1 0000	0 9999	0 9998	0 9996	0 9992	0 9917	0 9623	0 8929	0 7752	0 6188	0 4478	0 2902	0 1662
	7							1 0000	1 0000	1 0000	0 9999	0 9983	0 9891	0 9598	0 8954	0 7872	0 6405	0 4743	0 3145
	8										1 0000	0 9997	0 9974	0·9876	0 9597	0 9006	0 8011	0 6626	0 5000
	9											1 0000	0 9995	0 9969	0 9873	0 9617	0 9081	0 8166	0 6855
	10												0 9999	0 9994	0 9968	0 9880	0 9652	0 9174	0 8338
	11												1 0000	0 9999	0 9993	0 9970	0 9894	0 9699	0 9283
	12													1 0000	0 9999	0 9994	0 9975	0 9914	0 9755
	13														1 0000	0 9999	0 9995	0 9981	0 9936
	14															1 0000	0 9999	0 9997	0 9988
	15																1 0000	1 0000	0 9999
	16																		1 0000
	17																		—
18	0	0 8345	0 6951	0 5780	0 4796	0 3972	0 3283	0 2708	0 2229	0 1831	0 1501	0 0536	0 0180	0 0056	0 0016	0 0004	0 0001	—	—
	1	0 9862	0 9505	0 8997	0 8393	0 7735	0 7055	0 6378	0 5719	0 5091	0 4503	0 2241	0 0991	0 0395	0 0142	0 0046	0 0013	0 0003	0 0001
	2	0 9993	0 9948	0 9843	0 9667	0 9419	0 9102	0 8725	0 8298	0 7832	0 7338	0 4797	0 2713	0 1353	0 0600	0 0236	0 0082	0 0025	0 0007
	3	1 0000	0 9996	0 9982	0 9950	0 9891	0 9799	0 9667	0 9494	0 9277	0 9018	0 7202	0 5010	0 3057	0 1646	0 0783	0 0328	0 0120	0 0038
	4		1 0000	0 9998	0 9994	0 9985	0 9966	0 9933	0 9884	0 9814	0 9718	0 8794	0 7164	0 5187	0 3327	0 1886	0 0942	0 0411	0 0154
	5			1 0000	0 9999	0 9998	0 9995	0 9990	0 9979	0 9962	0 9936	0 9581	0 8671	0 7175	0 5344	0 3550	0 2088	0 1077	0 0481
	6				1 0000	1 0000	1 0000	0 9999	0 9997	0 9994	0 9988	0 9882	0 9487	0 8610	0 7217	0 5491	0 3743	0 2258	0 1189
	7							1 0000	1 0000	0 9999	0 9998	0 9973	0 9837	0 9431	0 8593	0 7283	0 5634	0 3915	0 2403
	8									1 0000	1 0000	0 9995	0 9957	0 9807	0 9404	0 8609	0 7368	0 5778	0 4073
	9											0 9999	0 9991	0 9946	0 9790	0 9403	0 8653	0 7473	0 5927
	10											1 0000	0 9998	0 9988	0 9939	0 9788	0 9424	0 8720	0 7597
	11												1 0000	0 9998	0 9986	0 9938	0 9797	0 9463	0 8811
	12													1 0000	0 9997	0 9986	0 9942	0 9817	0 9519
	13														1 0000	0 9997	0 9987	0 9951	0 9846
	14															1 0000	0 9998	0 9990	0 9962
	15																1 0000	0 9999	0 9993
	16																	1 0000	0 9999
	17																		1 0000
	18																		—

(*Continued*)

BINOMIAL DISTRIBUTION : PARTIAL SUMS – *Contd*

n	r ↓ \ p →	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
19	0	0·8262	0·6812	0·5606	0·4604	0·3774	0·3086	0·2519	0·2051	0·1666	0·1351	0·0456	0·0144	0·0042	0·0011	0·0003	0·0001	–	–
	1	0·9847	0·9454	0·8900	0·8249	0·7547	0·6829	0·6121	0·5440	0·4798	0·4203	0·1985	0·0829	0·0310	0·0104	0·0031	0·0008	0·0002	–
	2	0·9991	0·9939	0·9817	0·9616	0·9335	0·8979	0·8561	0·8092	0·7585	0·7054	0·4413	0·2369	0·1113	0·0462	0·0170	0·0055	0·0015	0·0004
	3	1·0000	0·9995	0·9978	0·9939	0·9868	0·9757	0·9602	0·9398	0·9147	0·8850	0·6841	0·4551	0·2631	0·1332	0·0691	0·0230	0·0077	0·0022
	4		1·0000	0·9998	0·9993	0·9980	0·9956	0·9915	0·9853	0·9765	0·9648	0·8556	0·6733	0·4654	0·2822	0·1500	0·0696	0·0280	0·0096
	5			1·0000	0·9999	0·9998	0·9994	0·9986	0·9971	0·9949	0·9914	0·9463	0·8369	0·6678	0·4739	0·2968	9·1629	0·0777	0·0318
	6				1·0000	1·0000	0·9999	0·9998	0·9996	0·9991	0·9983	0·9837	0·9324	0·8251	0·6655	0·4812	0·3081	0·1727	0·0835
	7						1·0000	1·0000	0·9999	0·9999	0·9997	0·9959	0·9767	0·9225	0·8180	0·6656	0·4878	0·3169	0·1796
	8								1·0000	1·0000	1·0000	0·9992	0·9933	0·9713	0·9161	0·8145	0·6675	0·4940	0·3238
	9											0·9999	0·9984	0·9911	0·9674	0·9125	0·8139	0·6710	0·5000
	10											1·0000	0·9997	0·9977	0·9895	0·9653	0·9115	0·8159	0·6762
	11												1·0000	0·9995	0·9972	0·9886	0·9648	0·9129	0·8204
	12													0·9999	0·9994	0·9969	0·9884	0·9658	0·9165
	13													1·0000	0·9999	0·9993	0·9969	0·9891	0·9682
	14														1·0000	0·9999	0·9994	0·9972	0·9904
	15															1·0000	0·9999	0·9995	0·9978
	16																1·0000	0·9999	0·9996
	17																	1·0000	1·0000
	18																		–
	19																		–
20	0	0·8179	0·6676	0·5438	0·4420	0·3585	0·2901	0·2342	0·1887	0·1516	0·1216	0·0388	0·0115	0·0032	0·0008	0·0002	–	–	–
	1	0·9831	0·9401	0·8802	0·8103	0·7358	0·6605	0·5869	0·5169	0·4516	0·3917	0·1756	0·0692	0·0243	0·0076	0·0021	0·0005	0·0001	–
	2	0·9990	0·9929	0·9790	0·9561	0·9245	0·8850	0·8390	0·7879	0·7334	0·6769	0·4049	0·2061	0·0913	0·0355	0·0121	0·0036	0·0009	0·0002
	3	1·0000	0·9994	0·9973	0·9926	0·9841	0·9710	0·9529	0·9294	0·9007	0·8670	0·6477	0·4114	0·2252	0·1071	0·0444	0·0160	0·0049	0·0013
	4		1·0000	0·9997	0·9990	0·9974	0·9944	0·9893	0·9817	0·9710	0·9568	0·8298	0·6296	0·4148	0·2375	0·1182	0·0510	0·0189	0·0059
	5			1·0000	0·9999	0·9997	0·9991	0·9981	0·9962	0·9932	0·9887	0·9327	0·8042	0·6172	0·4164	0·2454	0·1256	0·0553	0·0207
	6				1·0000	1·0000	0·9999	0·9997	0·9994	0·9987	0·9976	0·9781	0·9133	0·7858	0·6080	0·4166	0·2500	0·1299	0·0577
	7						1·0000	1·0000	0·9999	0·9998	0·9996	0·9941	0·9679	0·8982	0·7723	0·6010	0·4159	0·2520	0·1316
	8								1·0000	1·0000	0·9999	0·9987	0·9900	0·9591	0·8867	0·7624	0·5956	0·4143	0·2517
	9										1·0000	0·9998	0·9974	0·9861	0·9520	0·8782	0·7553	0·5914	0·4119
	10											1·0000	0·9994	0·9961	0·9829	0·9468	0·8725	0·7507	0·5881
	11												0·9999	0·9991	0·9949	0·9804	0·9435	0·8692	0·7483
	12												1·0000	0·9998	0·9987	0·9940	0·9790	0·9420	0·8684
	13													1·0000	0·9997	0·9985	0·9935	0·9786	0·9423
	14														1·0000	0·9997	0·9984	0·9936	0·9793
	15															1·0000	0·9997	0·9985	0·9941
	16																1·0000	0·9997	0·9987
	17																	1·0000	0·9998
	18																		1·0000
	19																		–
	20																		–

APPENDIX C

(*Clause* 7.1.1)

POISSON DISTRIBUTION : INDIVIDUAL TERMS

For m = 0.01 (0.01) 0.10, (0.05) 1.00 (0.1) 2 (0·2) 5

$$\frac{m^r e^{-m}}{r!}$$

r ↓ \ m →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
0	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139	0.9048	0.8607	0.8187	0.7788	0.7408	0.7047	0.6703	0.6376	0.6065	0.5769	0.5488	0.5220	0.4966	0.4724	0.4493	0.4274	0.4066
1	0.0099	0.0196	0.0291	0.0384	0.0476	0.0565	0.0653	0.0738	0.0823	0.0905	0.1291	0.1637	0.1947	0.2222	0.2466	0.2681	0.2869	0.3033	0.3173	0.3293	0.3393	0.3476	0.3543	0.3595	0.3633	0.3659
2		0.0002	0.0004	0.0008	0.0012	0.0017	0.0023	0.0030	0.0037	0.0045	0.0097	0.0164	0.0243	0.0333	0.0432	0.0536	0.0646	0.0758	0.0873	0.0988	0.1103	0.1217	0.1329	0.1438	0.1544	0.1647
3							0.0001	0.0001	0.0001	0.0002	0.0005	0.0011	0.0020	0.0033	0.0050	0.0072	0.0097	0.0126	0.0160	0.0198	0.0239	0.0284	0.0332	0.0383	0.0437	0.0494
4												0.0001	0.0001	0.0003	0.0004	0.0007	0.0011	0.0016	0.0022	0.0030	0.0039	0.0050	0.0062	0.0077	0.0093	0.0111
5																0.0001	0.0001	0.0002	0.0002	0.0004	0.0005	0.0007	0.0009	0.0012	0.0016	0.0020
6																					0.0001	0.0001	0.0001	0.0002	0.0002	0.0003

r ↓ \ m →	0.95	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.3867	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.3674	0.3679	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707	0.2438	0.2177	0.1931	0.1703	0.1494	0.1304	0.1135	0.0984	0.0850	0.0733	0.0630	0.0540	0.0462	0.0395	0.0337
2	0.1745	0.1839	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707	0.2681	0.2613	0.2510	0.2384	0.2240	0.2087	0·1929	0.1771	0.1615	0.1465	0.1323	0.1188	0.1063	0.0918	0.0842
3	0.0553	0.0613	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804	0.1966	0.2090	0.2176	0.2225	0.2240	0.2226	0.2186	0.2125	0.2046	0.1954	0.1852	0.1743	0.1631	0.1517	0.1404
4	0.0131	0.0153	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902	0.1082	0.1254	0.1414	0.1557	0.1680	0.1781	0.1858	0.1912	0.1914	0.1954	0.1944	0.1917	0.1875	0.1820	0.1755
5	0.0025	0.0031	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361	0.0476	0.0602	0.0735	0.0872	0.1008	0.1140	0.1264	0.1377	0.1477	0.1563	0.1633	0.1687	0.1725	0.1747	0.1755
6	0.0004	0.0005	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120	0.0174	0.0241	0.0319	0.0407	0.0504	0.0608	0.0716	0.0826	0.0936	0.1042	0.1143	0.1237	0.1323	0.1398	0.1462
7	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034	0.0055	0.0083	0.0118	0.0163	0.0216	0.0278	0.0348	0.0425	0.0508	0.0595	0.0686	0.0778	0.0869	0.0959	0.1044
8					0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009	0.0015	0.0025	0.0038	0.0057	0.0081	0.0111	0.0148	0.0191	0.0241	0.0298	0.0360	0.0428	0.0500	0.0575	0.0653
9									0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0011	0.0018	0.0027	0.0040	0.0056	0.0076	0.0102	0.0132	0.0168	0.0209	0.0255	0.0307	0.0363
10													0.0001	0.0002	0.0003	0.0005	0.0008	0.0013	0.0019	0.0028	0.0039	0.0053	0.0071	0.0092	0.0118	0.0147	0·0181
11															0.0001	0.0001	0.0002	0.0004	0.0006	0.0009	0.0013	0.0019	0.0027	0.0037	0.0049	0.0064	0.0082
12																	0.0001	0.0001	0.0002	0.0003	0.0004	0.0006	0.0009	0.0013	0.0019	0.0026	0.0034
13																				0.0001	0.0001	0.0002	0.0003	0.0005	0.0007	0.0009	0.0013
14																						0.0001	0.0001	0.0001	0.0002	0.0003	0.0005
15																									0.0001	0.0001	0.0002

APPENDIX D

(*Clause* 7.1.1)

POISSON DISTRIBUTION : PARTIAL SUMS

For m = 0.01 (0.01) 0.10 (0.05) 1.00 (0.1) 2 (0.2) 5

$$\sum_{x=0}^{r} \frac{m^x e^{-m}}{x!}$$

r ↓ \ m →	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
0	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139	0.9048	0.8607	0.8187	0.7788	0.7408	0.7047	0.6703	0.6376	0.6065	0.5769	0.5488	0.5220	0.4966	0.4724	0.4493	0.4274	0.4066
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962	0.9953	0.9898	0.9825	0.9735	0.9631	0.9513	0.9384	0.9246	0.9098	0.8943	0.8781	0.8614	0.8442	0.8266	0.8088	0.7907	0.7725
2		1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9995	0.9989	0.9978	0.9964	0.9945	0.9921	0.9891	0.9856	0.9815	0.9769	0.9717	0.9659	0.9595	0.9526	0.9451	0.9371
3							1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9992	0.9988	0.9982	0.9975	0.9966	0.9956	0.9942	0.9927	0.9909	0.9889	0.9865
4												1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989	0.9986	0.9982	0.9977
5																1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9997
6																					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

r ↓ \ m →	0.95	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.3867	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.7541	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.9287	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.2102	0.1851	0.1626	0.1425	0.1247
3	0.9839	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.9971	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.9995	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9999	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9361	0.9214	0.9049	0.8867	0.8666
8					1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9									1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9889	0.9851	0.9805	0.9749	0.9682
10													1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
11															1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9980	0.9971	0.9960	0.9945
12																	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9993	0.9990	0.9986	0.9980
13																			1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993
14																						1.0000	1.0000	0.9999	0.9999	0.9999	0.9998
15																								1.0000	1.0000	1.0000	0.9999
16																											0.9999
17																											1.0000

APPENDIX E

(*See Clause* 8.1.1)

LOGARITHMS OF FACTORIALS OF INTEGERS FROM 1 TO 100

MPLE SIZE n (1)	Log$_{10}$ n! (2)	SAMPLE SIZE n (1)	Log$_{10}$ n! (2)	SAMPLE SIZE n (1)	Log$_{10}$ n! (2)
1	0·000 0000	41	49·524 4289	81	120·763 2127
2	0·301 0300	42	51·147 6782	82	122·677 0266
3	0·778 1513	43	52·781 1467	83	124·596 1047
4	1·380 2112	44	54·424 5993	84	126·520 3840
5	2·079 1812	45	56·077 8119	85	128·449 8029
6	2·857 3325	46	57·740 5697	86	130·384 3013
7	3·702 4305	47	59·412 6676	87	132·323 8006
8	4·605 5205	48	61·093 9088	88	134·268 3033
9	5·559 7630	49	62·784 1049	89	136·217 6933
10	6·559 7630	50	64·483 0749	90	138·171 9358
11	7·601 1557	51	66·190 6450	91	140·130 9772
12	8·680 3370	52	67·906 6484	92	142·094 7650
13	9·794 2803	53	69·630 9863	93	144·063
14	10·940 4084	54	71·363 3180	94	146·036 3758
15	12·116 4996	55	73·103 6807	95	148 014 0994
16	13·320 6196	56	74·851 8687	96	149·996 3707
17	14·551 0685	57	76·607 7436	97	151·983 1424
18	15·806 3410	58	78·371 1716	98	153·974 3685
19	17·085 0946	59	80·142 0236	99	155·970 0037
20	18·386 1246	60	81·920 1148	100	157 970 0037
21	19·708 3439	61	83·705 5047		
22	21·050 7666	62	85·497 8964		
23	22·412 4944	63	87·297 2369		
24	23·792 7057	64	89·103 4169		
25	25·190 6457	65	90·916 3303		
26	26·605 6190	66	92·735 8742		
27	28·036 9828	67	94·561 9490		
28	29·486 1408	68	96·394 4579		
29	30·946 5388	69	98·233 3070		
30	32·423 6601	70	100·078 4050		
31	33·915 0218	71	101·929 6634		
32	35·420 1717	72	103·786 9959		
33	36·939 6857	73	105·650 3187		
34	38·490 1646	74	107·519 5505		
35	40·014 2326	75	109·394 6117		
36	41·570 5351	76	111·275 4253		
37	43·138 7369	77	113·161 9160		
38	44·718 5205	78	115·054 0106		
39	46·309 5851	79	116·951 6377		
40	47·911 6451	80	118·854 7277		

INDIAN STANDARDS
ON
QUALITY CONTROL, SAMPLING AND OTHER STATISTICAL TECHNIQUES

IS:

397 (Part I)-1972 Method for statistical quality control during production: Part I Control charts for variable (*first revision*)

397 (Part II)-1975 Method for statistical quality control during production: Part II Control charts for attributes and count of defects (*first revision*)

1548-1969 Manual on basic principles of lot sampling

2500 (Part I)-1973 Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

2500 (Part II)-1965 Sampling inspection tables: Part II Inspection by variables for percent defective

4905-1968 Methods for random sampling

5002-1969 Methods for determination of sample size to estimate the average quality of a lot or process

5420 (Part I)-1969 Guide on precision of test methods: Part I Principles and applications

5420 (Part II)-1973 Guide on precision of test methods: Part II Inter-laboratory testing

6200 (Part I)-1977 Statistical tests of significance: Part I μ t, normal and F tests (*first revision*)

6200 (Part II)-1977 Statistical tests of significance: Part II α^2-test (*first revision*)

7200 (Part I)-1974 Presentation of statistical data: Part I Tabulation and summarization

7200 (Part II)-1975 Presentation of statistical data: Part II Diagrammatic Representation of data

7300 1974 Methods of regression and correlation

7600-1975 Analysis of variance

7920 (Part I)-1976 Statistical vocabulary and symbols: Part I General statistical terms

7920 (Part II)-1976 Statistical vocabulary and symbols: Part II Terms used in sampling and process control

INTERNATIONAL SYSTEM OF UNITS (SI UNITS)

Base Units

Quantity	*Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic température	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Supplementary Units

Quantity	*Unit*	*Symbol*
Plane angle	radian	rad
Solid angle	steradian	sr

Derived Units

Quantity	*Unit*	*Symbol*	*Definition*
Force	newton	N	1 N = 1 kg.m/s^2
Energy	joule	J	1 J = 1 N.m
Power	watt	W	1 W = 1 J/s
Flux	weber	Wb	1 Wb = 1 V.s
Flux density	tesla	T	1 T = 1 Wb/m^2
Frequency	hertz	Hz	1 Hz = 1 c/s (s^{-1})
Electric conductance	siemens	S	1 S = 1 A/V
Electromotive force	volt	V	1 V = 1 W/A
Pressure, stress	pascal	Pa	1 Pa = 1 N/m^2

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